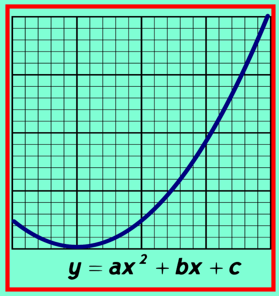
Math 125

Fall 2021

Lecture 31



Class QZ 25

Evaluate:

Always

$$\begin{vmatrix} 2 & 3 & -4 \\ 1 & -1 & 5 \\ 3 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} -1 & 5 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} + (-4)\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}$$

$$= 2(-1-10) - 3(1-15) - 4(2+3)$$

$$= -22 + 42 - 20 = \boxed{0}$$

Some Review:

1) write $x^{\frac{3}{5}}$ in radical notation.

$= \sqrt[5]{x^3}$

Index = 5

Radicand = x^3

2) write $\sqrt[8]{x^5}$ in Rational exponent.

$= x^{\frac{5}{8}}$

Index = 8

Radicand = x^5

Given $f(x) = \sqrt[2]{x-3}$

No index $\Rightarrow$ index = 2

Find

1) $f(3) = \sqrt{3-3}$

$= \sqrt{0} = \boxed{0}$

2) $f(4) = \sqrt{4-3}$

$= \sqrt{1} = \boxed{1}$

3) $f(7) = \sqrt{7-3}$

$= \sqrt{4}$

$= \boxed{2}$

4) $f(0) = \sqrt{0-3}$

$= \sqrt{-3}$

undefined

even index $\rightarrow$ Radicand ≥ 0

5) Domain of $f(x)$

Index = 2 $\rightarrow$ even index $\rightarrow$ Radicand ≥ 0

$x - 3 \geq 0$

$x \geq 3$

3

$\boxed{[3, \infty)}$

Simplify, write as one radical

$$\sqrt[5]{x^2} \cdot \sqrt[4]{x} = x^{\frac{2}{5}} \cdot x^{\frac{1}{4}}$$

$$x^m \cdot x^n = x^{m+n}$$

$$= x^{\frac{2}{5} + \frac{1}{4}}$$

$$\frac{2 \cdot 4}{5 \cdot 4} + \frac{1 \cdot 5}{4 \cdot 5}$$

$$= \frac{8}{20} + \frac{5}{20}$$

$$= x^{\frac{13}{20}}$$

$$= \sqrt[20]{x^{13}}$$

Index = 20

Radicand = x^{13}

Simplify, write as a single radical:

$$\frac{\sqrt{x}}{\sqrt[8]{x^3}} = \frac{x^{1/2}}{x^{3/8}}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$= x^{\frac{1}{2} - \frac{3}{8}}$$

$$\frac{1 \cdot 4}{2 \cdot 4} - \frac{3}{8} = \frac{4}{8} - \frac{3}{8}$$

$$= \frac{4-3}{8} = \frac{1}{8}$$

$$= x^{\frac{1}{8}}$$

$$= \sqrt[8]{x^1} = \boxed{\sqrt[8]{x}}$$

$$x^1 = x$$

Recall $\sqrt[n]{A}\sqrt[n]{B} = \sqrt[n]{AB}$

$$\frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \sqrt[n]{\frac{A}{B}}$$

Simplify: $\sqrt{5}\cdot\sqrt{10}$

$= \sqrt{5\cdot 10} = \sqrt{50}$

$= \sqrt{25\cdot 2} = \sqrt{25}\sqrt{2}$

$= \boxed{5\sqrt{2}}$

Simplify $\dfrac{\sqrt{125}}{\sqrt{5}} = \sqrt{\dfrac{125}{5}}$

$= \sqrt{25} = \boxed{5}$

Distribute & Simplify:

1) $\sqrt{6}\left(\sqrt{10} - \sqrt{3}\right) = \sqrt{6}\sqrt{10} - \sqrt{6}\sqrt{3}$

$= \sqrt{60} - \sqrt{18}$

$= \sqrt{4}\sqrt{15} - \sqrt{9}\sqrt{2}$

$= \boxed{2\sqrt{15} - 3\sqrt{2}}$

2) $2\sqrt{5}\left(3\sqrt{10} + 4\sqrt{5}\right)$

$= 6\sqrt{50} + 8\sqrt{25}$

$= 6\sqrt{25}\sqrt{2} + 8\sqrt{25}$

$= 6\cdot 5\sqrt{2} + 8\cdot 5$

$= \boxed{30\sqrt{2} + 40}$

FOIL & Simplify

1) $(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$

$=\sqrt{36}\cancel{-\sqrt{12}}\cancel{+\sqrt{12}}-\sqrt{4}$

$=6-2=\boxed{4}$ ✓

2) $(2\sqrt{5}+3)(2\sqrt{5}-3)$

$=4\sqrt{25}\cancel{-6\sqrt{5}}\cancel{+6\sqrt{5}}-9=4\cdot 5-9$

$=20-9=\boxed{11}$

Simplify

$(2\sqrt{3}+\sqrt{2})^2$

$=(2\sqrt{3}+\sqrt{2})(2\sqrt{3}+\sqrt{2})$

$=4\sqrt{9}+2\sqrt{6}+2\sqrt{6}+\sqrt{4}$

$=4\cdot 3+4\sqrt{6}+2$

$=12+4\sqrt{6}+2=\boxed{14+4\sqrt{6}}$

Hint:

$x^2=x\cdot x$

FOIL & Simplify

Simplify:

$(3\sqrt{5}+2\sqrt{3})(2\sqrt{5}-\sqrt{3})$

FOIL & Simplify

$=6\sqrt{25}-3\sqrt{15}+4\sqrt{15}-2\sqrt{9}$

$=6\cdot 5\quad +1\sqrt{15}\quad -2\cdot 3$

$=30+\sqrt{15}-6=\boxed{24+\sqrt{15}}$

Simplify:

$(4\sqrt{2}-\sqrt{5})^2$

Hint:

$x^2=x\cdot x$

Foil/Simplify

$=(4\sqrt{2}-\sqrt{5})(4\sqrt{2}-\sqrt{5})$

$=16\sqrt{4}-4\sqrt{10}-4\sqrt{10}+\sqrt{25}$

$=16\cdot 2\quad -8\sqrt{10}\quad +5$

$=32-8\sqrt{10}+5=\boxed{37-8\sqrt{10}}$

Rectangle

$P = 2L + 2W$

$A = LW$

$3\sqrt{5} - 2$

$3\sqrt{5} + 2$

Find Area & Perimeter.

$P = 2L + 2W = 2(3\sqrt{5}+2) + 2(3\sqrt{5}-2)$

$= 6\sqrt{5} + 4 + 6\sqrt{5} - 4 = \boxed{12\sqrt{5}}$ units

$A = LW = (3\sqrt{5}+2)(3\sqrt{5}-2)$

$= 9\sqrt{25} \cancel{-6\sqrt{5}} \cancel{+6\sqrt{5}} - 4$ units^2

$= 9 \cdot 5 - 4 = 45 - 4 = \boxed{41}$

$(\sqrt{x})^2 = x$

$(\sqrt[n]{x})^n = x$ $(x \geq 0)$

$(\sqrt[3]{x})^3 = x$

$(\sqrt[4]{x})^4 = x$

Simplify

$(\sqrt{2x+1})^2 = 2x+1$

$(\sqrt[3]{x-5})^3 = x-5$

$(\sqrt[4]{3x+5})^4 = 3x+5$

Solve

$\sqrt{x+4} = 6$

$\left(\sqrt{x+4}\right)^2 = \left(6\right)^2$

$x+4 = 36$

$x = 32$

$\{32\}$

Always isolate the radical.

Index = 2

Always check your solution

$\sqrt{32+4} = 6$

$\sqrt{36} = 6$

$6 = 6$ ✓

Solve

$\sqrt[3]{2x-1} + 5 = 0$

$\sqrt[3]{2x-1} = -5$

$\left(\sqrt[3]{2x-1}\right)^3 = \left(-5\right)^3$

$2x - 1 = -125$

$2x = -125 + 1$

$2x = -124$

$\boxed{x = -62}$

Isolate the radical

Raise both sides to the index power.

Always check

$x = -62$ ✓

$\{-62\}$

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Solve by Matrix Method:

$$\begin{cases} x + 2y + 3z = -5 \\ 2x + y + z = 1 \\ x + y - z = 8 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & -1 & 8 \end{array}\right]$$

$(-1)R1 + R3 \to R3$
$(-2)R1 + R2 \to R2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & -3 & -5 & 11 \\ 0 & -1 & -4 & 13 \end{array}\right]$$

$-R3 \to R3$
$R2 \leftrightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & 1 & 4 & -13 \\ 0 & -3 & -5 & 11 \end{array}\right]$$

$(3)R2 + R3 \to R3$
$(-2)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 21 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 7 & -28 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 21 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 7 & -28 \end{array}\right]$$

$R_3 \div 7 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 21 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$(-4)R3 + R2 \to R2$
$(5)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{array}\right] \quad \begin{array}{l} x = 1 \\ y = 3 \\ z = -4 \end{array}$$

$(1, 3, -4)$

Class QZ 26:

Solve by Cramer's rule for Y-only

$$\begin{cases} 4x - 3y = 14 \\ 3x - y = 3 \end{cases}$$

$$D = \begin{vmatrix} 4 & -3 \\ 3 & -1 \end{vmatrix} = 4(-1) - 3(-3) = -4 + 9 = \boxed{5}$$

$$D_y = \begin{vmatrix} 4 & 14 \\ 3 & 3 \end{vmatrix} = 4(3) - 3(14) = 12 - 42 = \boxed{-30}$$

$$y = \frac{D_y}{D} = \frac{-30}{5} \qquad \boxed{y = -6}$$